International Journal of Engineering, Science and Mathematics
Vol. 7, Special Issue 4(3), April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Continuous Variation of Critical Exponents Out of Universality Hypothesis

Prosenjit Sarkar*

Abstract

We have shown, from magnetization measurements, that the critical exponents, β, γ and δ, associated with ferromagnetic to paramagnetic phase transition in $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ ($0.5 \leq y \leq 1$) single crystals vary continuously with Nd concentration. The critical exponents for $y = 0.5$ crystal were come out to be ($\beta = 0.16$, $\gamma = 1.27$ and $\delta = 9.30$), which vary continuously with y and became ($\beta = 0.36$, $\gamma = 1.38$ and $\delta = 4.72$) for $y = 1$. The critical exponents for $y = 1$ are very close to the exponents of Heisenberg model for three dimension. The simultaneous variation of all three critical exponents violated both universality and weak universality hypothesis, suggesting a novel critical behaviour.

Keywords:

Phase transition;
Critical phenomena;
Scaling hypothesis;
Weak-universality;
Magnetoresistive manganites.

Author correspondence:

Prosenjit Sarkar,
Department of Physics, Serampore College
Serampore, Hooghly, West-Bengal – 712201, India
Email: psphysics1981@gmail.com

1. Introduction

The study of critical phenomena of phase transition is mainly based on two concepts: one is universality and another is scaling theory. The universality hypothesis states that the associated critical exponents are universal depending up to symmetries and space dimensionality, whereas the scaling theory relates the different critical exponents [1]. According to the renormalization group theory, the critical point is a fixed point governed by a unique set of relevant operators with critical exponents which are completely independent of irrelevant operators [2]. While a relevant perturbation may take the system to a new fixed point, the marginal operator brings a possibility of continuous variation of critical exponents. Although the concept of universality has been verified experimentally again and again, starting from early 40's to present, the continuous variation of exponents is rarely observed. One example is eight vertex model, where critical exponents vary continuously with the interaction parameter of the system [3]. This problem was solved by Suzuki by proposing a weak universality (WU) hypothesis, where critical exponents β and γ change continuously but their ratios and consequently $\delta = 1 + \gamma/\beta$ remain invariant [4]. This WU scenario has been observed in several theoretical models such as frustrated spin systems, interacting dimers, magnetic hard squares, Blume-Capel models, reaction diffusion systems, absorbing phase transitions, percolation models, fractal structures, as well as in some real systems [4]-[8]. The generic nature of the marginal interaction that leads to WU in all these different systems remains unclear.

It has been observed from the literature that most systems which show continuous variation of critical exponents obey WU, although there are few exceptions like criticality in Ising spin glass, micellar solutions, frustrated spin systems, strong coupling quantum electrodynamics etc [9]-[11]. Experimentally, the continuous evolution of critical exponents with doping has been observed in $URu_{2-x}Re_xSi_2$ ($0.2 \leq x \leq 0.6$) single crystals [12]. With decreasing x, both γ and δ vary linearly keeping β fixed. Recently, Fuch *et. al*. have observed the

* Department of Physics, Serampore College, Serampore, Hooghly, West-Bengal – 712201, India

linear variation of critical exponents in $Sr_{1-z}Ca_zRuO_3$ from ($\beta \approx 0.5$, $\gamma \approx 1$, $\delta \approx 3$) for $z = 0$ to ($\beta \approx 1$, $\gamma \approx 0.9$, $\delta \approx 1.6$) for $z = 0.6$ and they have predicted that the variation of exponents may be originating from quantum fluctuations associated with quantum phase transition at around $z = 0.7$ [13].

Non-conventional ferromagnetic (FM) to paramagnetic (PM) phase transition has also been observed in mixed valance manganites, $R_{1-x}A_xMnO_3$ (*R*: rare earth ions, *A*: alkaline earth ions) either as a discontinuous transition or a continuous transition with a set of critical exponents that does not belong to any known universality or the weak universality class [14]. In manganites, the nature of phases and transitions strongly depend on the bandwidth and local disorder which arises due to the size mismatch between *R* and *A* cations. A system with narrow bandwidth and large disorder such as $Sm_{1-x}Sr_xMnO_3$ shows a sharp first-order FM-PM transition for $x = 0.45 - 0.48$ [15]-[18]. The first-order transition is however extremely sensitive to external pressure, magnetic field (*H*), chemical substitution, oxygen isotope exchange, etc. - with the application of external and internal pressure (via chemical substitution) beyond a critical threshold, the transition becomes continuous.

In this paper, we report on a thermodynamic transition, *i.e.* FM to PM phase transition in $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ single crystals with $y = 0.5$ to 1.0, where all three critical exponents β, γ, and δ vary continuously. The variation of critical exponents is not in accordance with the weak universality hypothesis, suggesting a novel critical behaviour.

2. Research Method

The single crystals of $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ with $y = 0.5$, 0.6, 0.8 and 1.0 have been prepared by floating zone technique [19]. The starting materials, Sm_2O_3, Nd_2O_3, $SrCO_3$, and Mn_3O_4 were mixed in appropriate ratios, heated in air at ~1350 K for 20 h and then pulverized. This procedure was repeated for several times. The resultant powder was formed to cylindrical shape with the use of hydrostatic pressure to make a feed rod and then annealed. The apparatus used for crystal growth was the floating zone image furnace equipped with two halogen incandescent lamps. The feed and seed rods were rotated in opposite directions at 25 rpm. The molten zone was vertically scanned at a rate of 8–10 mm/h in oxygen atmosphere. The dc magnetization (*M*) measurements were done in a superconducting quantum interference device magnetometer. For *M* (*H*), the data were collected at 1 to 4 K intervals after stabilizing the temperature for about 15 min. All the measurements were performed with a slow sweep rate of temperature and magnetic field.

3. Results and Analysis

Figure 1 shows the temperature dependence of magnetization for $y = 0.5$, 0.8 and 1.0 at $H = 100$ Oe. *M* (*T*) shows typical ferromagnetic behaviour with T_C's ~192 K, 222K, 241K and 265K for $y = 0.5$, 0.6, 0.8 and 1.0, respectively. To reveal the nature of FM to PM phase transition, we measure a set of *M* (*H*) isotherms for different *y*. Here we present the *M* (*H*) curves for $y = 0.5$ sample in the figure 2 (a). The order of the FM transition can be checked from the slope of Arrott plots (M^2 vs *H/M*), which offers a criterion for determining whether such transition is first-order or second-order [20]. If the slope of M^2 vs *H/M* is positive then the transition is second-order and for a first-order transition, the slope becomes negative (Banerjee criterion) [21]. The Arrott plot for y = 0.5 is displayed in figure 2 (b). The observed positive slope indicates the FM transition is likely to be second-order.

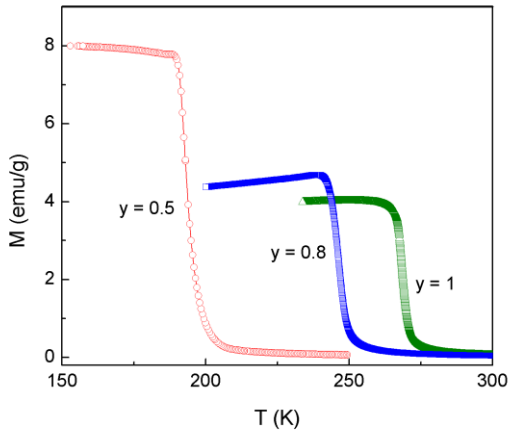

Figure 1. *Temperature (T) dependence of magnetization (M) of $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ single crystals for y = 0.5, 0.8 and 1.0 at a magnetic field, H = 100 Oe.*

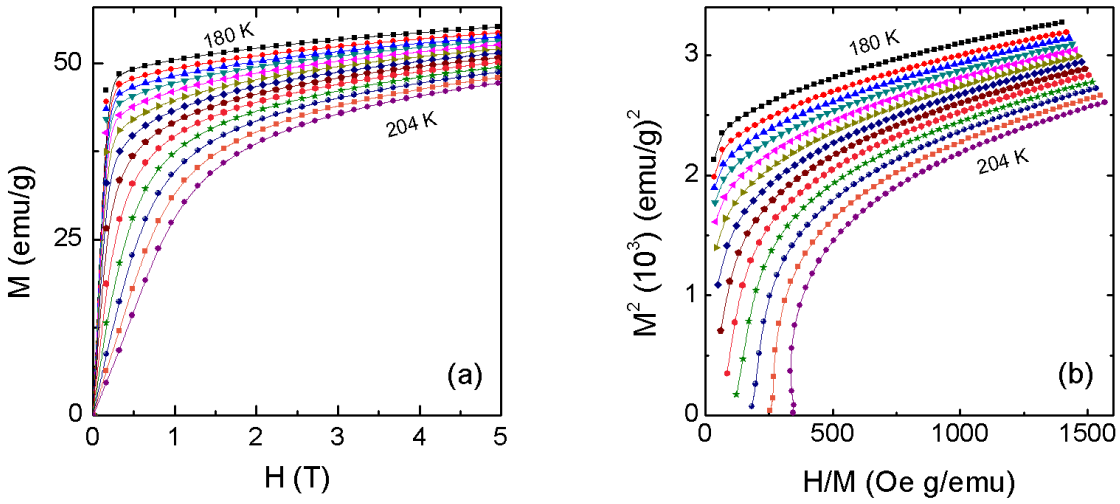

Figure 2. (a) *M (H) isotherms of* $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ *for y = 0.5 crystal between 180 K (top) and 204 K (bottom) in 2 K interval. (b) Arrott plot* (M^2 *vs H/M) isotherms for y = 0.5.*

Let us set notations by reminding that in absence of magnetic field, the spontaneous magnetization of the system vanishes as $M_S(0, \varepsilon) \sim (-\varepsilon)^{\beta}$ and the initial susceptibility diverges as $\chi_0(0, \varepsilon) \sim (-\varepsilon)^{\gamma}$ as the critical point is approached, *i.e.*, when $(T/T_C - 1) \equiv \varepsilon \rightarrow 0$. Again at $T = T_C$, the magnetization varies as $M(H, T_C) \sim H^{1/\delta}$. From figure 2 (b), it is clear that the isotherms in the Arrott plot are nonlinear, making it difficult to identify the critical isotherm ($T = T_C$) that passes through the origin. Therefore, we use the modified Arrott plot method, based on the Arrott-Noaks equation of state which is given by [20]

$$\left(\frac{H}{M}\right)^{1/\gamma} = a\varepsilon + bM^{1/\beta},$$

where a and b are non-universal constants. The correct choice of β and γ can make the isotherms of $M^{1/\beta}$ versus $(H/M)^{1/\gamma}$ a set of parallel straight lines with one unique critical isotherm that passes through the origin, which is shown in figure 3 for $y = 0.5$ and the self consistency is achieved for the values $\beta = 0.16$, $\gamma = 1.30$.

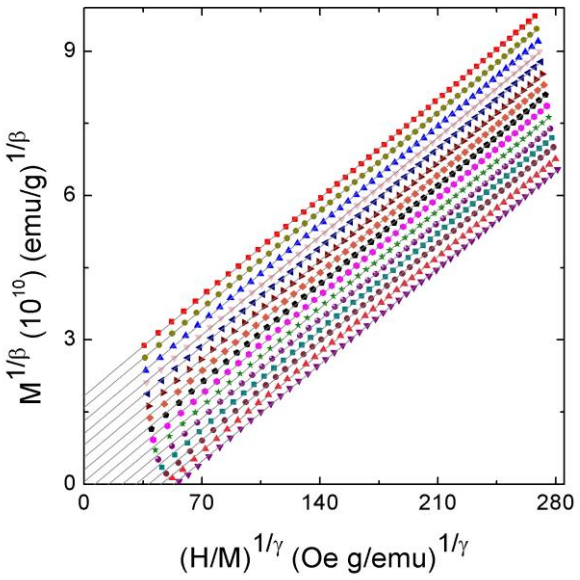

Figure 3. Modified Arrott plot ($M^{1/\beta}$ *vs* $(H/M)^{1/\gamma}$) *isotherms of* $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ *for y = 0.5 with* $\beta = 0.16$, $\gamma = 1.30$ *and* $T_C = 192$ *K.*

The isotherm T = 192 K almost passes through the origin. From the intercepts of these parallel straight lines on $M^{1/\beta}$ and $(H/M)^{1/\gamma}$ axes, we obtain M_S and χ_0^{-1} for different temperatures which are shown in the figures 4 (a) and 4 (b), respectively. The best power-law fit gives β = 0.16, γ = 1.27 and T_C = 192.3 K.

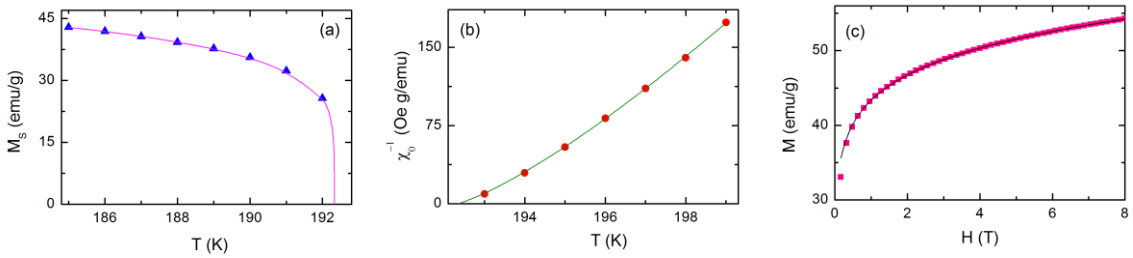

Figure 4. y = 0.5: Temperature dependence of (a) spontaneous magnetization (M_S) and (b) inverse initial susceptibility (χ_0^{-1}). (c) M (H) curve at $T = T_C$.

Another critical exponent δ can be determined from M (H) isotherm at $T = T_C$. For this purpose, we use the M-H curve at T = 192 K (for y = 0.5), the nearest one to the critical isotherm and the polynomial fitting gives δ = 9.30 [see figure 4 (c)]. Following the same method, we determine the critical exponents and T_C's for y = 0.6, 0.8 and 1.0, which are listed in Table 1.

Table 1. Critical exponents of $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ for different y

y	β	γ	δ
0.5	0.16	1.27	9.30
0.6	0.23	1.30	6.31
0.8	0.31	1.32	5.14
1.0	0.36	1.38	4.72

The accuracy of the critical exponents and T_C can be checked by comparing the data with scaling hypothesis, which predicts that M (H, ε) is a universal function of H and ε [1],

$$M(H,\varepsilon) = (\varepsilon)^{\beta} f_{\pm}[H\varepsilon^{-(\gamma+\beta)}],$$

where f_+ is for $T > T_C$ and f_- is for $T < T_C$. All the data points for $T > T_C$ are expected to fall on f_+, whereas the data points for $T < T_C$ will be on f_-.

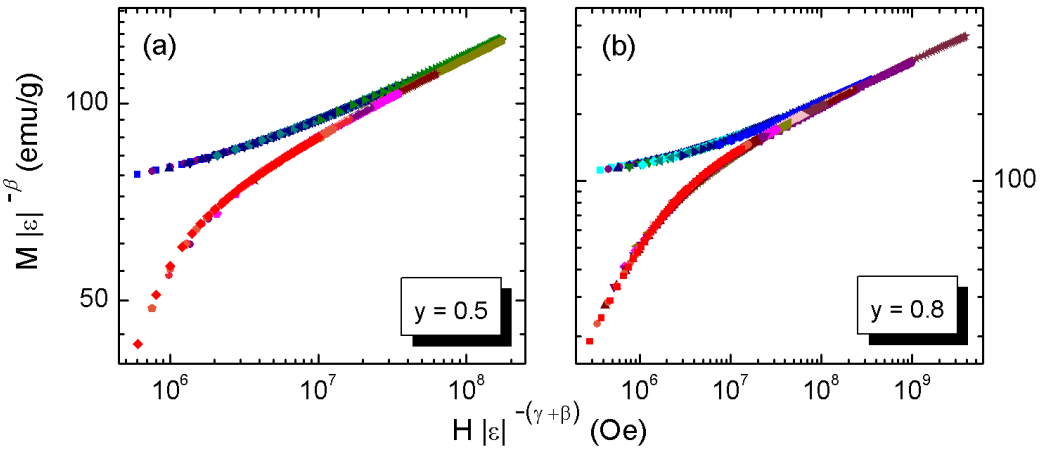

Figure 5. Scaling plots (on log-log scale) of $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ for y = 0.5 and 0.8.

We plot this scaling curve for all samples but for clarity, the curves for y = 0.5 and 0.8 are presented in figure 5 (a) and 5 (b), respectively. It is clear from the figure that the data points over the entire range of the variable fall on two branches of the curve depending on the sign of ε. The validity of the scaling hypothesis further confirms that the values of the exponents and T_C are unambiguous and self-consistent.

4. Conclusion

In conclusion, we have studied the critical phenomenon in $(Sm_{1-y}Nd_y)_{0.52}Sr_{0.48}MnO_3$ single crystals with $0.5 \leq y \leq 1$. The values of critical exponents (β, γ, δ) measured for $y = 1$ are in accordance with three dimensional Heisenberg universality class, whereas the same for $y = 0.5, 0.6, 0.8$ are far from any known universality class. All these exponents vary continuously with Nd concentration, y, but they seem to obey the standard scaling laws following a single equation of state. The variation of critical exponents is not new to critical phenomena, but most examples satisfy the weak universality hypothesis, where β and γ vary but δ remains fixed. The simultaneous variation of all three critical exponents is beyond weak universality hypothesis, a novel phenomenon.

Acknowledgements

The author would like to thank Prof. P. Mandal, Prof. P. K. Mohanty, Dr. N. Khan and Dr. A. Midya of SINP, Kolkata for discussions and Mr. A. Pal for technical assistance.

References

[1] Stanley, H. E., "Scaling, universality, and renormalization: three pillars of modern critical phenomena," *Rev. Mod. Phys.*, vol. 71, pp. S358–S366, 1999.
[2] Wilson, K. G. and Kogut, J., "The renormalization group and the ε expansion," *Physics Reports*, vol. 12, pp. 75–199, 1974.
[3] Baxter, R. J., "Eight-vertex model in lattice statistics," *Phys. Rev. Lett.*, vol. 26, pp. 832–833, 1971.
[4] Suzuki, M., "New universality of critical exponents," *Prog. Theor. Phys*., vol. 51, pp. 1992–1993, 1974.
[5] Queiroz, S. L. A. de., "Scaling behavior of a square-lattice Ising model with competing interactions in a uniform field," *Phys. Rev. E*, vol. 84, pp. 031132, 2011.
[6] Jin, S., Sen, A. and Sandvik, A. W., "Ashkin-Teller criticality and pseudo-first-order behavior in a frustrated Ising model on the square Lattice," *Phys. Rev. Lett.*, vol. 108, pp. 045702, 2012.
[7] Pearce, P. A. and Kim, D., "Continuously varying exponents in magnetic hard squares," *J. Phys. A: Math. Gen*., vol. 20, pp. 6471–6485, 1987.
[8] Malakis, A., Berker, A. N., Hadjiagapiou, I. A. and Fytas, N. G., "Strong violation of critical phenomena universality: Wang-Landau study of the two-dimensional Blume-Capel model under bond randomness," *Phys. Rev. E*, vol. 79, pp. 011125, 2009.
[9] Newman, T. J., "Continuously varying exponents in reaction-diffusion systems," *J. Phys. A: Math. Gen*., vol. 28, pp. L183–L190, 1995.
[10] Fisher, M. E., "Long-range crossover and nonuniversal exponents in micellar solutions," *Phys. Rev. Lett*., vol. 57, pp. 1911–1914, 1986.
[11] Kondo, K. I., "Critical exponents, scaling law, universality and renormalization group flow in strong coupling QED," *Int. J. Mod. Phys. A*, vol. 6, pp. 5447–5466, 1991.
[12] Butch, N. P. and Maple, M. B., "Evolution of critical scaling behavior near a ferromagnetic quantum phase transition," *Phys. Rev. Lett*., vol. 103, pp. 076404, 2009.
[13] Fuchs, D. *et al.*, "Critical scaling analysis of the itinerant ferromagnet $Sr_{1-x}Ca_xRuO_3$," *Phys. Rev. B*, vol. 89, pp. 174405, 2014.
[14] Tokura, Y., "Critical features of colossal magnetoresistive manganites," *Rep. Prog. Phys*., vol. 69, pp. 797–851, 2006.
[15] Demkó, L. *et al.*, "Multicritical end point of the first-order ferromagnetic transition in colossal magnetoresistive manganites," *Phys. Rev. Lett*., vol. 101, pp. 037206, 2008.
[16] Sarkar, P. *et al.*, "Pressure induced critical behavior of ferromagnetic phase transition in Sm-Nd-Sr manganites," *Phys. Rev. Lett*., vol. 103, pp. 057205, 2009.
[17] Sarkar, P. *et al.*, "Role of external and internal perturbations on the ferromagnetic phase transition in $Sm_{0.52}Sr_{0.48}MnO_3$," *Phys. Rev. B*, vol. 79, pp. 144431, 2009.
[18] Khan, N., Sarkar, P., Midya, A., Mandal, P. and Mohanty, P. K., "Continuously varying critical exponents beyond weak universality," *Scientific Reports*, vol. 7, pp. 45004, 2017.
[19] Mandal, P., Bandyopadhyay, B. and Ghosh, B., "Resistivity anomaly in the vicinity of a structural phase transition in $La_{1-x}Sr_xMnO_3$," *Phys. Rev.* B., vol. 64, pp. 180405(R), 2001.
[20] Arrott, A. and Noakes, J. E., "Approximate equation of state for nickel near its critical temperature," *Phys. Rev. Lett.*, vol. 19, pp. 786–789, 1967.
[21] Banerjee, B. K., "On a generalised approach to first and second order magnetic transitions," *Phys. Lett*., vol. 12, pp. 16-17, 1964.